INTRODUCTION

A cement rotary kiln is a distributed parameter process system which has a highly complex behaviour due to chemical reactions. It is one of the key equipment in the cement industry to convert calcineous raw materials to cement clinkers. Raw meal for cement production is a mixture of predetermined proportions of limestone, silica, and small quantities of alumina and iron oxide. The chemistry of making cement was studied in some detail (Bougie, 1955; Lua, 1956). The raw meal is first fed to pre-calciner where about 60-80% of calcination takes place. Thereafter, the partially calcinated feed is charged into the rotary kiln. The primary function of a rotary kiln is to provide a high-temperature environment to drive solid-solid
Figure 1. A simplified scheme of a rotary cement kiln.

solid–liquid reactions for clinker formation. The energy required to carry out the reaction is provided by a counter-current burning of fuel. In the beginning section of a kiln, starting from solid entrance, the calcination of raw material is completed. Thereafter, the solids undergo solid–solid reactions as they move forward. Then, the solid enters a higher temperature zone where it melts to form a liquid phase. The final reactions take place in this phase. So that it causes formation of coating over the refractory lining in the remaining part of the kiln. In this higher temperature section, called burning zone, the refractory of the kiln is under severe damage because of high temperature. Moreover, the high-coating thickness creates serious problem for the flow of solids along the kiln. Therefore, the thickness of the coating in the burning zone is momentous because of the protection of the refractory lining from damaging which increases the life of the refractory and movement of feed through the kiln.

Methods based on measuring the thermal radiation from the shell surface were conventional for checking and estimating the coating thickness and lining in the rotary cement kilns. In this way, a rapid scanning device for temperature measurement of the shell is used in combination with a computer which produces a visual display of the temperature profile (Wulff, 1993).

A method developed for estimating the coating thickness was the transient kiln model (Boklab, 1994). In this method, the inside temperature of the kiln was considered as the average temperature of gas and solid. After measuring the shell temperature, the coating thickness was estimated by considering two resistant nodes between the inside and outside temperatures. The results were not reliable because there was no calculation for temperature profiles inside the kiln. Moreover, the heat transfer between the shell and the environment was calculated by a simple equation. The relation between the shell temperature and the coating thickness in the burning zone were reported as monographs (Pisters and Beeke, 1985; Schubin et al., 1985). In these monographs, the coating thickness was curved versus air and shell temperature. These graphs were only valid for a limited range of operating conditions. The transient method could be a good starting point to develop rigorous and accurate coating thickness estimators, but as mentioned before, calculating the inside profile temperature has a direct effect on the exact prediction of the coating thickness.

Although the rotary cement kiln is a key piece of equipment, attempts for developing computational models to simulate cement kilns were few. Recently, some researchers have been done to develop computational fluid dynamics (CFD) based models to simulate rotary cement kiln (Mastorakos et al., 1999; Lu et al., 2004). But, it was not an applicable method for the coating thickness estimation in practice, because of the considerable calculation time to integrate the scanned shell temperature with the kiln model. Moreover, some attempts were made to develop kinetic base models for the kiln (Mujumdar and Ranade, 2006; Mujumdar et al., 2007). Such models were shown promising capabilities in capturing the overall behaviour of cement kilns. However, most of the reported models did not account for the estimation of the coating thickness.

In this paper, based on the developed previous dynamic model (Spang, 1972) a rigorous kinetic based steady-state model derived from heat and mass balances for the inside temperature profile of the kiln is developed. Since the Spang approach was valid for coal burners, it could not be used efficiently for gas and oil fuel burners; therefore, to model the flame, a plug flow model (Gorog et al., 1983) is used in this work. Furthermore, the heat transfer coefficients in the previous work (Spang, 1972) were presumed constant, while in this approach, these coefficients are calculated according to available equations (Schwarzkopf, 1974; Kaminski, 1977; Gorog et al., 1982).

After calculating the temperature profile of gas, solid material and wall, the formed coating thickness in the burning zone is estimated by considering two resistant nodes between the inside calculated temperature and the outside scanned temperature.

**QUALITATIVE DESCRIPTION OF A CEMENT KILN**

A simplified model of a cement kiln is shown in Figure 1. It is basically a drum approximately 50–120 m long (for modern kilns) and 3.5–6.5 m in diameter. The length of the kiln mainly depends on how long it takes to heat up the raw material to “clinkering temperature” of around 1570°C. In modern designs of the plant, a cyclone pre-heater is used to raise the input solid temperature to shorten the length of the kiln.

The raw material fed into the kiln contains calcium carbonate (CaCO₃), silica (SiO₂), shale (Al₂O₃), and iron ore (Fe₂O₃). These are ground into very fine powder and mixed according to the type of cement being made. Upon heating by the hot gases, various reactions occur in different sections of the kiln, as shown in Table 1.

| Table 1. Common reactions occur in a cement kiln (Spang, 1972) |
|---------------------------------|-----------------|------------------|
| Temp (K) | Reactions | Heat change |
| 373 | Evaporation | Endothermic |
| >1100 | Calcination CaCO₃ → CaO + CO₂ | Endothermic |
| 1573 | Clinkering reaction 2CaO + SiO₂ → Ca₂SiO₄ | Exothermic |
| 3CaO + Al₂O₃ → Ca₃A | Exothermic |
| >1673 | 4CaO + Al₂O₃ + Fe₂O₃ + C → Ca₄Al₂O₆ | Endothermic |
| CaO + C₆S → C₆S | Endothermic |
Table 2. Composition of ground material fed to pre-heater

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Mass %</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO₂</td>
<td>14.76</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>2.89</td>
</tr>
<tr>
<td>Fe₂O₃</td>
<td>2.38</td>
</tr>
<tr>
<td>CaO</td>
<td>42.12</td>
</tr>
<tr>
<td>Loss of Ignition</td>
<td>1.41</td>
</tr>
<tr>
<td>SO₃, K₂O, Na₂O</td>
<td>0.84</td>
</tr>
</tbody>
</table>

**INDUSTRIAL CEMENT KILN**

The length, inside and outside diameters of the studied cement kiln were 66.7, 3.95, and 4.35 m, respectively. The thickness of the refractory lining was 19 cm, which could be considered uniform through the burning zone. The cylinder slope was 4% to facilitate the axial displacement of the solid bed, moving towards the discharge end while the hot gases circulate in the opposite current mode. The raw materials used in the clinker production were presented in Table 2. They were ground and mixed together, then they were sent into the pre-heater to be partially calcinated. Finally, the solids from the calciner were fed into the rotary kiln. The length of the burning zone was 35 m, extended from the burner to the middle of the kiln. Therefore, the coating thickness was necessary to be estimated only for this region. The fuel of the kiln was natural gas, but in winter, fuel oil No. 6 was used by a dual fuel burner as a complementary fuel.

**MATHEMATICAL MODEL**

Various processes occurring in the rotary cement kiln which is needed to be adequately considered while developing its mathematical model. Clinker is obtained from burning raw material in a rotary kiln that required energy for calcination and other reactions as well as melt formation is provided by the hot free board gases. Several chemical reactions take place in the solid bed and the melt formation causes an internal coating on the surface of the refractory in the burning zone. The system is highly nonlinear due to complex heat/mass transfer.

**Modelling Assumptions**

During the development of the model, it was tried to keep the structure as simple as possible. Some of these assumptions could be implemented without increasing the complexity and decreasing the accuracy of the model. These were as follows:

- The inside and outside diameters of the kiln were constant.
- The specific and reaction heats were independent of temperature and they were constant along the axial direction.
- Conduction in gases and solid materials in the axial direction of the wall was neglected.
- Coefficients of convection and emissivity were independent of temperature and position.
- The height and speed of solid materials were constant at each cross-section of the kiln.
- The transported solids by gas stream were not included in the model.
- Reaction rates were determined by Arrhenius law.
- The H₂O and CO₂ entering the gas stream from combustion reaction were neglected in the mass balances of these components.

Kiln Modelling

For determining the required temperature profiles inside the kiln, it was not necessary using a complex dynamic model. It was due to decrease of the model complications and to alleviate the problem of long run time calculations. Therefore, a steady-state one-dimensional model was developed for calculating the temperature profiles in the kiln.

**Energy balance**

For performing energy balance, an averaged overall wall temperature was used in the model. The required Equations (1)-(3) were written for gas, solid, and wall, respectively (Spang, 1972). All coefficients and constants were summarised in Appendix A.

\[
\Delta C_{p} \rho \frac{\partial T}{\partial z} = \beta_1 (T_w - T_k) + \beta_3 (T_s - T_k) + Q_{comb} 
\]

(1)

Solid:

\[
\Delta C_{p} \rho \frac{\partial T}{\partial z} = \beta_1 (T_k - T_s) + \beta_3 (T_s - T_k) + A_0 Q_c 
\]

(2)

Wall:

\[
\beta_1 (T_k - T_w) + \beta_3 (T_s - T_w) + B_4 (T_s - T_w) = 0 
\]

(3)

Heat of reaction:

\[
Q_c = \frac{\rho_c}{(1 + A_k + F_1 + S_1)} \left[ -\Delta H_1 k_1 \xi - \Delta H_2 R_2 - \Delta H_3 k_2 S(C)^2 - \Delta H_4 k_4 C_\beta \right] 
\]

(4)

(\beta_1, \beta_3, \beta_4) are nonlinear functions of temperatures, convection, and radiation heat transfer coefficients, and geometry which can be calculated by Equations (5)-(8).

Heat transfer coefficient between the gases and the inside wall is as follows:

\[
\beta_1 = 1.7507 r_i \frac{T_i}{p} [f_1 + 1.73] \times 10^9 (1 - f_c) \varepsilon \sigma \left( T_i^4 + T_\beta^{-4} \right) \left( T_\beta + T_i \right) 
\]

(5)

Heat transfer coefficient between the gases and the solid is as follows:

\[
\beta_2 = 3.4314 r_i \sin \left( \frac{p}{2} \right) [f_2 + 1.73] \times 10^9 (1 - f_c) \varepsilon \sigma \left( T_2^4 + T_\beta^4 \right) \left( T_2 + T_i \right) 
\]

(6)

Heat transfer coefficient between the wall and the solid is as follows:

\[
\beta_3 = r_1 (2\pi - p) [f_3 + 1.73 \times 10^8 h_s w(T_s^4 + T_\beta^4) (T_\beta + T_s)] 
\]

(7)

Heat transfer coefficient between the outside wall and the ambient temperature is as follows:

\[
\beta_4 = 2\pi f_4 r_2 
\]

(8)

For the calculation of heat transfer coefficient between the wall and the environment (outside air), Spang used the Equation (8). It was a rough description of heat transfer coefficient between the shell surface and the environment. Since f_w was constant through the kiln, \beta_4 was not sensitive to conditions in the longitudinal
direction. In order to increase the accuracy of the model, the heat-transfer coefficient of the outer shell was assumed to be the sum of convective \( h_{\text{conv}} \rightarrow a \) and radiative \( h_{\text{rad}} \rightarrow a \) heat transfer coefficients given by (Schwarzkopf, 1974; Kaminski, 1977), respectively.

\[
h_{\text{conv}} \rightarrow a = 0.11 k \frac{n_{a} \Delta T_{a}}{D} (0.5 Re_{a}^{0.8} \text{Re}^{2} + Gr \text{Re}^{0.15})
\]

(9)

\[
h_{\text{rad}} \rightarrow a = C \frac{e_{a} \sigma T_{a}^{4}}{T_{a}^{4}}
\]

(10)

where \( C \) in Equation (10) was:

\[
C = \left\{ 1 + \frac{T_{a}}{T_{a}} + \left( \frac{T_{a}}{T_{a}} \right)^{2} + \left( \frac{T_{a}}{T_{a}} \right)^{3} \right\}
\]

(11)

As it can be understood from Equation (8), \( h_{\text{conv}} \rightarrow a \) is dependent on temperature because \( Pr, Re, \text{ and } Gr \) can be strongly affected by environmental conditions. Therefore, to improve the model, at first, the shell temperature along the kiln length should be recorded from shell temperature scanner. Then, the dimensionless groups in the Equation (9) were calculated according to the film temperature (average value of the shell and air temperatures) along the kiln length. So, the value of \( h_{\text{conv}} \rightarrow a \) could be calculated in each longitudinal cross-section along the kiln. Because the shell temperature varies with time, this variation instantaneously affected the model so the heat transfer phenomenon in the model was rather a quasi-dynamic. This approach allowed considering the variations in convective heat transfer coefficient, dependent both on time and longitudinal distance.

**Mass balance**

The mass balances, Equations (12) and (13) were written for the involved 10 solid components in the reactions, occurring in the cement kilns and gas component (\( CO_{2} \)):

\[
R_{i} = \frac{\partial C_{i}}{\partial t}
\]

(12)

\[
R_{f} = \frac{\partial C_{f}}{\partial t}
\]

(13)

with: \( H_{2}O, CaCO_{3}, SiO_{2}, Al_{2}O_{3}, Fe_{2}O_{3}, CaO, C_{1}S, C_{2}S, C_{3}A, C_{4}AF; \text{ and } f: CO_{2}. \)

The expressions for kinetic rates (\( K \)) were presented in Appendix A. In addition, related kinetic parameters were given in Table A1.

**Flame model**

As mentioned before, a plug flow model was used for the cement kiln (Gorog et al., 1983). In this model, the flame zone was divided into "n" slices of equal size. The thickness of each slice is equal to step-size of mathematical equation solver which is discussed later.

The number of slices was calculated as:

\[
n = \frac{F_{k}}{\text{Simulation step-size}}
\]

(14)

The overall flame length, \( F_{k} \), was obtained from the Beer equation (Gorog et al., 1983):

\[
F_{k} = 6 \omega_{a} (1 + AF^{*}) \left( \frac{\rho_{e}}{\rho_{e}} \right)^{0.5} \left( \frac{\rho_{e}}{\rho_{m}} \right)^{0.1}
\]

(15)

In the Gorog work, the cement kiln was equipped with a double coaxial-type burner (Gorog et al., 1982):

\[
\rho_{e} = \frac{m_{f} + m_{pa}}{m_{f} + m_{pa}}
\]

(16)

\[
d_{o} = \frac{m_{f} + m_{pa}}{[(Gr + G_{bo}) \rho_{e}]^{0.5}}
\]

(17)

\[
AF^{*} = \frac{AF_{o} - m_{pa}}{m_{f}}
\]

(18)

According to the previous investigations (Pearce, 1973; Ruddland, 1967; Gorog et al., 1982) the prediction of Equation (15) was in broad agreement with the flames and it was found to be accurate to within 20% of actual values.

Values of \( m_{f} \) and \( m_{pa} \) could be recorded from the instruments of the cement kiln. The value of \( m_{pa} \) was calculated by knowing \( m_{f} \) and excess air which was between 10% and 15% for the burner.

The required parameters for the studied fuel were presented in Table A2. Also, the \( G_{f} \) and \( G_{pa} \), in Equation (17) were instantaneous flow rate of fuel and primary air, respectively, calculated by the diameters of coaxial burner which were presented in Table A3.

Now, the \( Q_{\text{cmax}} \) in Equation (1) was calculated by the following expression:

\[
Q_{\text{cmax}} = \frac{m_{f} \times LHV + m_{b} \times G_{bo} \times (T_{bo} - T_{1}) + m_{pa} \times G_{pa} \times (T_{pa} - T_{2})}{\text{Simulation step-size}}
\]

(19)

**Simulation**

Mass and energy balance equations integrated with the flame model were the set of differential and algebraic equations solved by MATLAB software. Because of stiffness of equations especially in the burning zone, the ODE15S solver was used. After solving the model, the profile of the wall temperature was sent to the coating estimation sub-program. Since that discretisation of the kiln length was done appropriately by ODE solver, the wall temperature within the step sizes was known. For example in Figure 2, the cross-section of the kiln from 44 to 45 m was selected. To solve the model equations, the ODE meshed the kiln length to 1100 step sizes. Therefore, the meshing size for the program became 0.06 m. In Figure 2a, it was observed that for each mesh number there was a related wall temperature. Therefore, two generated dimensional matrices showed the temperature of the wall versus mesh points. The similar matrix was produced for kiln length, presented in Figure 2b. The temperature of the inside wall of the kiln versus kiln length could also be generated in a similar matrix which was sent to the coating estimation subprogram as wall temperature nodes.

**COATING ESTIMATION MODELLING**

To estimate the coating thickness the following assumptions were used:

1. The average value of coating conductivity was assumed to be equal to 0.75 W/m²/K (Schubin et al., 1985).
2. The conductivity of the refractory could be estimated by Equation (20) which was correlated from the experimental data.
Figure 2. (a) Wall temperature versus mesh numbers. (b) Wall temperature versus kiln length.

Figure 3. (a) Wall layers in burning zone of cement kiln. (b) Resistances of layers.

5- The number of scanned shell temperature points for a complete rotation of the kiln was thirty. The temperature of each calculation point through the kiln was assumed an average mathematical value of all points.

Resistant layers between the inner wall surface and the environment can be shown in Figure 3.

Heat flow in cylindrical coordinates (no heat generation) is:

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial Z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \tag{21}
\]

With the following assumptions, the Equation (21) can be simplified as:

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = 0 \tag{22}
\]

Assumptions:

(1) The heat transfer through layers of the kiln wall was steady state.
(2) Heat flow via conduction in z-direction was neglected.
(3) In each longitudinal segment, the wall temperature in \( \phi \)-direction was lumped.

Therefore, according to Figure 3a, the following boundary conditions can be defined (Figure 3b). Coating layer:

\( @r = r_w, T = T_w, \quad \& \quad @r = r_c, T = T_c \)

Refractory layer:

\( @r = r_c, T = T_c, \quad \& \quad @r = n_b, T = T_b \)

given by the refractory vendor for the magnetite-fired brick type:

\[ k_{\text{vcl}} = 3195.5 \times T_{\text{vcl}}^{0.922} \tag{20} \]

3- The conductivity of metallic shell was considered equal to 45 W/m²/K which was close to carbon steel alloy.

4- Since it was impossible to track variations in the conductivity of refractory lining during process, it was assumed that its value was constant equal to the start of run (Table 3). This assumption could be one of the sources for the gross error of the model prediction.

<table>
<thead>
<tr>
<th>Kiln length (m)</th>
<th>Thickness of refractory (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>66.7–60</td>
<td>25</td>
</tr>
<tr>
<td>60–43</td>
<td>22</td>
</tr>
<tr>
<td>45–40</td>
<td>20</td>
</tr>
<tr>
<td>40–30</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 3. Thickness of refractory in the burning zone.
Table 4. Variables required for estimating coating thickness

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source of values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{\text{env}}$</td>
<td>Heat transfer coefficient (kiln to environment)</td>
<td>$h_{\text{env}}$ is obtained by summation of Equations (9) and (10) Described in Coating Estimation Modelling Section</td>
</tr>
<tr>
<td>$\Delta Z$</td>
<td>Simulation step-size</td>
<td>Assumption No. 1 in Flame Length Section</td>
</tr>
<tr>
<td>$k_c$</td>
<td>Conductivity of coating layer</td>
<td>Assumption No. 2 in Flame Length Section</td>
</tr>
<tr>
<td>$k_b$</td>
<td>Conductivity of refractory material</td>
<td>Mentioned in Industrial Cement Kiln Section</td>
</tr>
<tr>
<td>$r_c$</td>
<td>Inner refractory surface radius</td>
<td>Mathematical model in Coating Estimation Modelling Section</td>
</tr>
<tr>
<td>$T_{\text{c}}$</td>
<td>Coating surface temperature</td>
<td>Summation of $r_c$, refractory thickness presented in Table 5</td>
</tr>
<tr>
<td>$T_{\text{s}}$</td>
<td>Inner shell surface radius</td>
<td>Mentioned in Industrial Cement Kiln Section</td>
</tr>
<tr>
<td>$T_{\text{sh}}$</td>
<td>Outer radius of kiln</td>
<td>Assumption No. 3 in Flame Length Section</td>
</tr>
<tr>
<td>$k_{\text{sh}}$</td>
<td>Conductivity of shell body</td>
<td>Measured by T-Scanner</td>
</tr>
<tr>
<td>$T_{\text{sh}}$</td>
<td>Shell outside surface temperature</td>
<td>Measured by user</td>
</tr>
<tr>
<td>$T_{\text{s}}$</td>
<td>Air temperature</td>
<td></td>
</tr>
</tbody>
</table>

Metal layer:

@ $r = r_b$, $T = T_b$, @ $r = r_{\text{sh}}$, $T = T_{\text{sh}}$

Therefore, by implementing the above boundary conditions and Equation (22), the following equations are obtained:

$$Q_{\text{loss}} = \frac{2\pi \Delta Z k_c}{\ln \left( \frac{r_b}{r_c} \right)} (T_b - T_c) \quad (23)$$

$$Q_{\text{loss}} = \frac{2\pi \Delta Z k_b}{\ln \left( \frac{r_{\text{sh}}}{r_b} \right)} (T_{\text{sh}} - T_b) \quad (24)$$

$$Q_{\text{loss}} = \frac{2\pi \Delta Z k_{\text{sh}}}{\ln \left( \frac{r_{\text{sh}}}{r_b} \right)} (T_{\text{sh}} - T_{\text{sh}}) \quad (25)$$

$$Q_{\text{loss}} = 2\pi \Delta Z r_b h_{\text{env}} \left( T_{\text{sh}} - T_{\text{s}} \right) \quad (26)$$

$$\Omega = r_c - r_b \quad (27)$$

$\Omega$ in Equation (27) is the thickness of coating. To estimate $\Omega$, at first $r_b$ and $r_{\text{sh}}$ should be determined. The required values for using Equations (23)–(27) were presented in Table 4.

To estimate the coating thickness in each step ($\Delta Z$), at first to find inside wall temperature of the kiln ($T_w$), Equations (1)–(19) were solved simultaneously. Then, by using Equations (26), (25), and (24), $Q_{\text{loss}}$, $T_b$, and $T_{\text{sh}}$ could be calculated, respectively. Finally, the estimation of the coating thickness ($\Omega$) was possible by calculating $r_b$ from Equation (23) and implementing of that in Equation (27).

In the simulation code, both inside wall temperature of the kiln ($T_w$) and kiln shell surface temperature ($T_{\text{sh}}$) were 2D matrixes of temperatures versus mesh points along the kiln. It could be concluded that the thermal resistant Equations (23)–(26) could be developed for the corresponded elements of matrixes therefore the profile of the coating thickness alongside the kiln length was calculated.

To compare the estimated and measured product values, absolute average error (AAE) from the following equation were calculated:

$$\text{AAE} = \frac{\sqrt{\sum (\Omega_{\text{meas}} - \Omega_{\text{estim}})^2}}{N_t} \quad (28)$$

### RESULTS AND DISCUSSION

The period of the refractory maintenance and the repairing time for the studied cement kiln in this study was about 6 months. Therefore, during 2 years, three actual data sets were gathered which are presented in Table 5. To obtain data, before shutting down the kiln and stopping fuel injection, the process variables were recorded. Then after cooling the kiln, the thicknesses of coating in various positions were measured.

#### Flame Length

The flame length of the studied cases (Table 5) was presented in Table 6. There was not any instrument to measure flame length exactly. But from the industrial records, the flame length was estimated around 5–8 m. In the case of using fuel oil, the flame length was longer. The results of simulation confirmed these records. Moreover, it could be concluded that the flame length is directly dependent on fuel flow rate.

Table 5. Operating conditions of studied cases

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air temperature (°C)</td>
<td>15</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Wind velocity (m/s)</td>
<td>30</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Inlet secondary air temperature (°C)</td>
<td>910</td>
<td>824.9</td>
<td>1200</td>
</tr>
<tr>
<td>Flow rate of Secondary air (m³/h)</td>
<td>5000</td>
<td>5200</td>
<td>5800</td>
</tr>
<tr>
<td>Fuel gas flow rate (m³/h)</td>
<td>3980</td>
<td>3200</td>
<td>5200</td>
</tr>
<tr>
<td>Fuel oil flow rate (kg/h)</td>
<td>0</td>
<td>4805</td>
<td>0</td>
</tr>
<tr>
<td>Inlet solid temp. (°C)</td>
<td>844</td>
<td>830</td>
<td>826</td>
</tr>
<tr>
<td>Feed rate (ton/h)</td>
<td>228.2</td>
<td>248.2</td>
<td>198.2</td>
</tr>
<tr>
<td>Speed of rotation (rpm)</td>
<td>1.8</td>
<td>2.6</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Table 6. Flame length for cases under study

<table>
<thead>
<tr>
<th></th>
<th>Flame length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>5.13</td>
</tr>
<tr>
<td>Case 2</td>
<td>9.64</td>
</tr>
<tr>
<td>Case 3</td>
<td>8.74</td>
</tr>
</tbody>
</table>
Temperature Profiles

Figures 4–6 showed steady-state temperature profiles of gas, solid material, and wall for cases 1, 2, and 3, respectively which were generated by the model. The shape of these curves was in agreement with typical experimental ones (Spang, 1972; Witsel et al., 2000). Besides, it could be seen that the temperature pick points of gas, solid and wall curves were at the end of the flame. This phenomenon was reported in the aforementioned research papers.

Scanner Temperature and Coating Thickness

In Figures 7, 9, and 11, the shell temperature measured by the scanner was presented for three studied cases, respectively. The points in these curves were spots on the shell surface that their temperatures were measured by the scanner. The position of them was always constant through the axial direction. By interpolation between these points, the temperature in any required point on the surface could be correlated.

In Figures 8, 10, and 12, for each case, the estimated coating thickness was compared with the experimental data. In most cross-sectional points of the kiln, the model showed an acceptable compatibility with the experimental data especially in the flame zone where the thickness of coating was more important than the other sections. The AAE for the case 1, 2, and 3 were 3.26, 2.82, and 2.21 cm, respectively. But in most of the points, the model estimated higher values than the experimental data. The main source of this error was instability of the created coating before the flame. The unstable coating layers in this region were prone to collapse during shutting down and cooling procedures. Another source of the error was assuming constant coating conductivity at 0.75 W/m²/K for all sections which could be changed from 0.5 to 1 W/m²/K.

As it could be found from Figures 7, 9, and 11, there were sharp variations of shell temperature, measured by the scanner. The reason for these phenomena was alteration of the coating thickness. Figures 8, 10, and 12 illustrated when there was ascending in shell temperature, there was proportional descending in coating thickness and vice versa.

Because the coating thickness from 10 to 15 cm is ideal for the protection of refractory in all areas of the burning zone (Bokalan, 1994), the curves proved that to maintain this coating thickness, the shell temperature should be held between 190 and 220°C. The lower temperature is a barricade for movement of solids along the kiln and the upper value is harmful for the refractory layer.
Figure 8. Comparison of actual coating thickness (△) with model estimation (■) for case 1.

Figure 9. Shell temperature profile for case 2.

Figure 11. Shell temperature profile for case 3.

Figure 12. Comparison of actual coating thickness (△) with model estimation (■) for case 3.

CONCLUSIONS

Because of the importance of the coating thickness in cement kilns to protect the refractory lining, an integrated model to estimate the coating thickness in cement kilns was developed. At first, a mathematical steady-state model was formulated to estimate the temperature profile of the inner surface of the wall of cement kiln. After that, by the calculated temperature profile along the kiln length and the measured temperature profile of the outer surface, the coating thickness was estimated by using a heat transfer resistant model in adjacent to cylindrical layers. The comparison of model results and three sets of data which were gathered from an industrial kiln, confirmed that the model had good capability to calculate the coating thickness.

Besides, it was concluded that the difference between the estimated values by model with experimental data could be from the coating conductivity in the burning zone and the breaking down of unstable coating during shutting down and cooling process.

Finally, the curves demonstrated that to have an acceptable coating thickness from the viewpoint of solid flow along the kiln and refractory protection, the shell temperature between 190 and 220°C was satisfactory.
NOMENCLATURE

- $A$: area of gas at given cross section (m$^2$)
- $A_b$: area of solid at given cross section (m$^2$)
- $A_w$: area of wall at given cross section (m$^2$)
- $A_F$: stoichiometric air–fuel ratio (mass basis)
- $A_F^*$: stoichiometric air–fuel ratio for a double coaxial burner (mass basis)
- $C_{p,g}$: specific heat of gas products 1173.82 (J/kg/K)
- $C_{p,s}$: specific heat of solid 1089.97 (J/kg/K)
- $C$: calcium oxide (CaO)
- $C_{a,g}$: 2CaO·SiO$_2$
- $C_{a,A}$: CaO·Al$_2$O$_3$
- $C_{a,AF}$: 4CaO·Al$_2$O$_3$·Fe$_2$O$_3$
- $d_b$: equivalent burner diameter
- $E$: depending on the subscript, activation energy of reactions (J/kmol)
- $f_1$: coefficient of conduction—gas to wall 22.71 (W/m$^2$/K)
- $f_2$: coefficient of conduction—solid to gas 22.71 (W/m$^2$/K)
- $f_3$: coefficient of conduction—wall to solid 22.71 (W/m$^2$/K)
- $f_4$: coefficient of conduction—wall to outside air 22.71 (W/m$^2$/K)
- $F_1$: initial value of kg Fe$_2$O$_3$/kg CaO
- $F_l$: flame length (m)
- $h_{r,a}$: fraction of radiation 0.6757
- $h_{r,sh}$: heat transfer coefficient of shell surface to air (W/m$^2$/K)
- $k$: depending on the subscript, Arrhenius coefficient of reactions (1/h)
- $k_{b,b}$: conductivity of the lining break or refractory (W/m$^2$/K)
- $k_{c,r}$: conductivity of coating (W/m$^2$/K)
- $k_{sh}$: conductivity of shell body (W/m$^2$/K)
- $K$: depending on the subscript, reaction rate of chemicals
- $LHV$: low heat value of fuel (J/kg)
- $m_{f}$: mass flow rate of fuel (kg/s)
- $m_{pa}$: mass flow rate of primary air (kg/s)
- $m_{wa}$: mass flow rate of secondary air (kg/s)
- $M$: depending on the subscript, molecular weight of chemicals
- $N$: number of measured points in each case
- $p$: angle subtended by surface of solid (3$\pi$/2)
- $Q_c$: heat generated by chemical reaction (W/m$^3$)
- $Q_{h,g}$: heat released by fuel combustion (J/s)
- $Q_{h,sh}$: flow heat passed from inside kiln to outside (J/s)
- $r_1$: inside radius of kiln 1.975 (m)
- $r_2$: outside radius of kiln 2.175 (m)
- $R$: ideal gas constant 8.314 (J/moL/K)
- $r_b$: radial distance from kiln center to refractory surface (m)
- $r_c$: radial distance from kiln center to wall surface (m)
- $r_w$: radial distance from kiln center to coating surface (m)
- $r_{sh}$: radial distance from kiln center to shell surface (m)

$S_i$: initial value of kgSiO$_2$/kgCaO
$T_o$: air temperature (K)
$T_b$: temperature of lining break or refractory (K)
$T_c$: temperature of coating (K)
$T_h$: gas temperature (K)
$T_s$: solid temperature (K)
$T_{ref}$: reference temperature (K)
$T_{sec}$: temperature of secondary air (K)
$T_{sh}$: temperature of shell surface (K)
$V_g$: velocity of gas (m/s)
$V_a$: velocity of solid (m/s)
$\beta_{b}, \beta_{c}, \beta_{s}$: heat transfer coefficients (W/K)
$\rho_g$: density of gas 0.25 (kg/m$^3$)
$\rho_s$: density of solid 890 (kg/m$^3$)
$\Delta Z$: solver step-size (m)
$\Omega$: coating thickness (m)

ACKNOWLEDGEMENTS

The authors wish to thank Eshd-س Sanat Company and Darab Cement Industry of Iran for their financial support.

APPENDIX A

Linear Speeds

$v_f = \frac{m_{f}}{A_b \cdot \rho_s}$
$v_a = \frac{(m_g + m_{sa})}{A_b \cdot \rho_s}$

Area Coefficients

$A_b = \frac{r_1^2}{2}(\pi - \sin \beta)$
$A_c = \frac{r_1^2}{2}(2\pi - \beta + \sin \beta)$

$A_{sh} = 2\pi(r_e^2 - r_1^2)$
$h = 1 - \frac{2h_{r,a}\sin(\frac{\beta}{2})}{2\pi - \beta}$

Kinetic Rates

$R_h = \begin{cases} -k_{b,b} \frac{M_{CaO}}{M_{CaO}} & \frac{M_{CaO}}{M_{CaO}} < 0.1 \\ -k_{b,b} \frac{M_{CaO}}{M_{CaO}} & \frac{M_{CaO}}{M_{CaO}} > 0.1 \end{cases}$

$R_{CaCO_3} = -M_{CaCO_3} \cdot k_{c,b} \cdot C_{CaCO_3}$

$R_{CaS} = -M_{CaS} \cdot k_{c,b} \cdot C_{CaO} \cdot C_{CaS}$
\[ R_{C3S} = \frac{M_{C3S}}{2M_{CaO}} \cdot k_{C3S} \cdot C_{CaO}^2 \cdot C_{Al2O3} = \frac{M_{C3S}}{2M_{CaO}} \cdot k_{C3S} \cdot C_{CaO} \cdot C_{Al2O3} \cdot C_{C3S} \]

\[ R_{C2A} = \frac{M_{C2A}}{3M_{CaO}} \cdot k_{C2A} \cdot C_{CaO} \cdot C_{Al2O3} \cdot C_{C2A} \]

\[ R_{C4AF} = \frac{M_{C4AF}}{4M_{CaO}} \cdot k_{C4AF} \cdot C_{CaO}^4 \cdot C_{Al2O3} \cdot C_{C4AF} \]

\[ R_{C3S} = \frac{M_{C3S}}{2M_{CaO}} \cdot k_{C3S} \cdot C_{CaO}^2 \cdot C_{Al2O3} \]

\[ R_{C2A} = \frac{M_{C2A}}{3M_{CaO}} \cdot k_{C2A} \cdot C_{CaO} \cdot C_{Al2O3} \cdot C_{C2A} \]

\[ R_{C4AF} = \frac{M_{C4AF}}{4M_{CaO}} \cdot k_{C4AF} \cdot C_{CaO}^4 \cdot C_{Al2O3} \cdot C_{C4AF} \]

\[ k_t = A_0 \exp \left( \frac{-E_t}{R T_s} \right) \]

with \( t = \text{CaO}, \text{C}_{3}S, \text{C}_{2}S, \text{C}_{3}A, \text{C}_{4}AF, \text{H}_{2}O, \text{CO}_2 \) (Tables A1–A3).

### Table A1. Kinetic parameters of reactions in cement kiln

<table>
<thead>
<tr>
<th>Reaction Rate</th>
<th>( A_0 ) (1/h)</th>
<th>( E_t ) (kJ/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{C2A} )</td>
<td>1.94 \times 10^4</td>
<td>42.08 \times 10^4</td>
</tr>
<tr>
<td>( R_{C3S} )</td>
<td>4.53 \times 10^4</td>
<td>805.8 \times 10^4</td>
</tr>
<tr>
<td>( R_{C4AF} )</td>
<td>1.33 \times 10^4</td>
<td>256.19 \times 10^4</td>
</tr>
<tr>
<td>( R_{C3S} )</td>
<td>4.11 \times 10^4</td>
<td>193.31 \times 10^4</td>
</tr>
<tr>
<td>( R_{C4AF} )</td>
<td>8.38 \times 10^4</td>
<td>185.16 \times 10^4</td>
</tr>
<tr>
<td>( R_{C3S} )</td>
<td>8.33 \times 10^4</td>
<td>194.10 \times 10^4</td>
</tr>
</tbody>
</table>

### Table A2. Combustion properties of studied fuel

<table>
<thead>
<tr>
<th>Fuel type</th>
<th>Heating value (kJ/kg)</th>
<th>Air-to-fuel ratio (kg/kg)</th>
<th>Standard density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural gas</td>
<td>5.327 \times 10^7</td>
<td>16.97</td>
<td>0.77</td>
</tr>
<tr>
<td>No. 6 fuel oil</td>
<td>4.221 \times 10^7</td>
<td>13.26</td>
<td>910</td>
</tr>
</tbody>
</table>

### REFERENCES


